

JEL Code :

Keyword :

1 Introduction and Overview

Risk, Uncertainty, and Profit

tial

“

essen-

the part

ipso facto

of market participants

both

ex ante

fully

in terms of fundamentals

2 Characterizing Knightian Uncertainty

ex ante

! + !
! 1 2 { } 1 2 "2.
!

ex ante

! "
+ 1 2
+ € + #

where the end-points of the interval τ_{t+1} in (4) are given by

$$\begin{aligned} & \tau_{t+1}^L = \tau_t + \frac{\sigma}{\sqrt{1+\sigma^2}} \left(\frac{\sigma}{\sqrt{1+\sigma^2}} \right) \\ & \tau_{t+1}^H = \tau_t + \frac{\sigma}{\sqrt{1+\sigma^2}} \left(\frac{\sigma}{\sqrt{1+\sigma^2}} \right) \end{aligned}$$

This specifies the probability distribution in terms of $\{\tau_{t+1}\}$ conditional on τ_t and for the given time- value of t .

$$\begin{aligned} & \tau_{t+1}^L = \tau_t + \frac{\sigma}{\sqrt{1+\sigma^2}} \left(\frac{\sigma}{\sqrt{1+\sigma^2}} \right) \\ & \tau_{t+1}^H = \tau_t + \frac{\sigma}{\sqrt{1+\sigma^2}} \left(\frac{\sigma}{\sqrt{1+\sigma^2}} \right) \end{aligned}$$

2.1 Knightian Uncertainty Constraints

$$\tau_{t+1}^L \leq \tau_t \leq \tau_{t+1}^H$$

by:

$$+1 \in \{+1, +1, +1, +1, \dots, - + \# \& \dots, + + \# \& \dots, - + \dots\}$$

where $- +, 0 \leq \# \leq 1$ and the initial condition is $- \leq 1 \leq +$.

+1

+1

- +

#

any any

$$\rightarrow \infty \quad + \quad \begin{matrix} \# & \$ \\ - & + \end{matrix}$$

Lemma 2

Assumption 2 Given the value μ_{t+1} , μ_t can take any value within the interval given by

$$\mu_t \in [\mu_{t+1} - \alpha, \mu_{t+1} + \alpha] \quad \text{where } \alpha \geq 0$$

where $\alpha \geq 0$, $0 \leq \alpha \leq 1$ and the initial condition is $\mu_0 \leq \mu_1 \leq \mu_2$.

$$\alpha \geq 1$$

Lemma 3 The KU constraint (20) implies that viewed from time t , for $t \geq 1$,

$$\mu_t \in [\mu_{t+1} - \alpha, \mu_{t+1} + \alpha] \quad \text{and} \quad \mu_{t+1} \in [\mu_{t+2} - \alpha, \mu_{t+2} + \alpha]$$

and that the end-points of the intervals satisfy the following intertemporal monotonicity property:

$$\mu_{t+1} \leq \mu_{t+2} \quad \text{and} \quad \mu_{t+1} \geq \mu_{t+2}$$

where $\alpha \geq 0$ and $0 \leq \alpha \leq 1$.

0

$$\mu_t \in [\mu_{t+1} - \alpha, \mu_{t+1} + \alpha]$$

Lemma 4 Lemmas 2 and 3 specify the end-points of the stochastic interval, in (17) within which μ_t lies, when viewed from time- t , in terms of μ_{t+1} and a set of

$$\mu_{t+1}$$

exogenously fixed parameters $(\alpha, \beta, \gamma, \delta, \epsilon, \zeta)$

$$\begin{aligned}
 & + \alpha + \beta \left(\frac{\gamma}{1} \right) + \delta \left(\frac{\epsilon}{1} \right) + \zeta \\
 & + \alpha + \beta \left(\frac{\gamma}{1} \right) + \delta \left(\frac{\epsilon}{1} \right) + \zeta
 \end{aligned}$$

where α, β, γ and δ, ϵ are given in (22) and (8) respectively.

4.3 The Knightian Uncertainty Expectation of Dividends

$$\begin{aligned}
 & + \\
 & +
 \end{aligned}$$

constraints in (8) and (21) that:

$$\begin{aligned} & (\quad) \\ & (+) \quad + \quad \binom{\%}{1} \quad + \quad \binom{\%}{1} \quad + \end{aligned}$$

where $(!)$, and $+$

well

but economists as

5.1 REH's Implementation of Muth's Hypothesis

+ - + -

precisely every

$\mathcal{F} (+)$ $\mathcal{F} (+)$

+

$$\begin{aligned}
& + \sum_{i=1}^n \alpha_i \geq 1 \quad \text{any } \alpha_i \geq 0 \\
& \mathcal{F}(t) = \mathcal{F}(t) \quad \mathcal{F}(t) = \mathcal{F}(t) \quad \mathcal{F}(t) = \mathcal{F}(t) \quad \% \\
& \mathcal{F}(t) = \mathcal{F}(t) \\
& \% \quad ()
\end{aligned}$$

Remark 2 *The representation in (34) illustrates the key implication of assuming that the process driving outcomes, such as dividends, does not undergo unforeseeable change. Applying Muth's hypothesis in such models, as REH does, constrains representations of participants' forecasts of dividends at each $t + \Delta t$, $t + 2\Delta t$, to be uniform, in the sense that every market participant is assumed to select exactly the same quantitative forecast of dividends in making his demand and supply decisions. Moreover, REH fully determines the representation of the so-called "representative agent's" forecasts in terms of the model's coefficients, (α_i) and the moments of its stochastic innovations, ϵ_{it} .*

5.2 Relating Participants' Forecasts of Dividends to Earnings under Knightian Uncertainty

+

+

$\mathcal{F} (+) \quad \% + \quad \in \quad (+)$

$(+)$

! +

%

$\% + \in ! + \# ! + ! \$ \# + \$$

+

+

$\mathcal{F} (+)$

% +

! +

$\mathcal{F} (+1) \quad \mathcal{F} (+1)$

% +
% +

$\mathcal{F} (+) \quad \% +$

$\mathcal{F}_t^i(d_{t-k})$

$KE_t(d_{t-k})$

% + € ! + # ! + ! + \$
% + ! +

specific

$$\mathcal{F} (+1) \quad \mathcal{F} (+1)$$

$$\mathcal{F} (+1)$$

$$\mathcal{F} (+1) \in$$

$$(+1$$

Moreover, \mathcal{I} is given by,

$$\mathcal{I} = \frac{\%}{1} \$ (+)$$

where

$$\begin{aligned} \frac{\%}{1} \$ + \left(\frac{\%}{1} + \right) \\ \frac{\%}{1} \$ + \left(\frac{\%}{1} + \right) \end{aligned}$$

and the model-implied bounds $+ , + , + ,$ and $+ +$ are specified in (9) and (22).

Remark 3 In the special case in which $\# \# 0$, in (9) and (22), the Knightian uncertainty about future $+ +$ and $+ +$

$$\% \quad ' \quad \mathcal{F} (\quad +1) \quad \mathcal{F} (\quad +1)$$

ambiguity

$$\begin{aligned} & ! \quad \$ (\% + ') ! \quad + \$ (! \% + ! ') \quad -_1 \\ ! \quad - \quad -_1 \quad & \$ (\% + ') ! \\ & -_1 \quad \$ (! \% + ! ') \quad -_1 \end{aligned}$$

8 Reconciling Model Consistency with Behavioral Evidence

Remark 6 *As we mentioned in the Introduction, we follow the convention in referring to "fundamental" and "non-fundamental" factors as exogenous variables that, respectively, an economist includes and does not include in his specification of div-*

' 1
7

$$\mathcal{F}(\quad +1) \quad \% \quad \mathcal{F}(\quad +1) \quad ' \quad '$$

$$\% \quad (\quad) \quad ' \quad \frac{''(\quad (\quad))}{1-''(\quad)}$$

represented with a probabilistic rule, such as Markov switching.

diverse autonomous

8.4 KUH: Representing the Role of Market Sentiment in Consistent Models

et al.

et al.

8.4.1 Modifying Bounds for Representations of Participants' Forecasts

Hypothesis 1

Remark 10 *Representations in (66) and (67) highlight the essential role of Muth's hypothesis in building intertemporal models under Knightian uncertainty. Imposing consistency within a KUH model enables an economist to represent and test the influence of non-fundamental factors (market sentiment) on aggregate outcomes (stock prices).*

8.4.2 Market Sentiment in Participants' Forecast Revisions

representations of participants' forecasts are partly autonomous: they are not completely determined in terms of the model's KU parameters, α_{t-1}^i , β_{t-1}^i , γ_{t-1}^i , δ_{t-1}^i , its coefficients at time $t-1$, and t , and the moments of its stochastic innovations.

$$\begin{aligned} \alpha_{t-1}^i &= \alpha_{t-1}^i + \epsilon_{t-1}^i, & \beta_{t-1}^i &= \beta_{t-1}^i + \epsilon_{t-1}^i, \\ \gamma_{t-1}^i &= \gamma_{t-1}^i + \epsilon_{t-1}^i, & \delta_{t-1}^i &= \delta_{t-1}^i + \epsilon_{t-1}^i \end{aligned}$$

8.4.3 Representations of Participants' Forecast Revisions

Hypothesis 2

(i) If the market is optimistic at time t , that is, if $\alpha_{t-1}^i > 1$, and $\beta_{t-1}^i < 1$ and $\gamma_{t-1}^i > \delta_{t-1}^i$, then

$$\begin{aligned} \alpha_{t-1}^i &\in (\alpha_{t-1}^i, 1) \\ \beta_{t-1}^i &\in (1, \beta_{t-1}^i) \end{aligned}$$

(ii) If the market is pessimistic at time t , that is, if $\alpha_{t-1}^i < 1$, and $\beta_{t-1}^i > 1$ and $\gamma_{t-1}^i < \delta_{t-1}^i$, then

$$\begin{aligned} \alpha_{t-1}^i &\in (1, \alpha_{t-1}^i) \\ \beta_{t-1}^i &\in (\beta_{t-1}^i, 1) \end{aligned}$$

$$\begin{aligned} (\alpha_{t-1}^i - \beta_{t-1}^i) &> (\gamma_{t-1}^i - \delta_{t-1}^i) \end{aligned}$$

$$\begin{aligned} & \text{! } \in \# \\ & \text{! } \in \# \\ & \text{! } \in \# \end{aligned}$$

Lemma 5 *If β_{-1} in (1) and β_{-1} in (14) the constraints $\beta_{-1} \leq \beta_{+1}$ and $\beta_{-1} \leq \beta_{+1}$ in Hypothesis 2 are satisfied. Analogously if β_{-1} and β_{-1} , the constraints $\beta_{-1} \leq \beta_{+1}$ and $\beta_{-1} \leq \beta_{+1}$ are satisfied.*

Remark 12 *Lemma 5 reveals the theoretical importance of behavioral finance's empirical findings that non-fundamental factors exert an autonomous, significant influence on how market participants behave.*

ex ante

Annual Review of Financial Economics,

Journal of Financial Economics,

Handbook of the Economics of Finance

Journal of Economic Literature,

The Journal of Finance,

Journal of Applied Econometrics

The Methodology and Practice of Econometrics

American Economic Review

The American Economic Review

Risk, Uncertainty and Profit

Econometrica

Journal of Economic Theory

Appendix (For online publication)

A Proofs of Lemmas and Theorems

Proof of Lemmas 1 and 3.

$$+ \geq$$

$$\begin{aligned}
+2 &\geq \frac{+1 +2 +1}{+1 +2} \frac{+!}{+!} +2 + \frac{+1 +2 +!}{+! +2} +2 \\
&\geq \binom{+}{1} +2 \binom{+}{1}
\end{aligned}$$

$$\begin{aligned}
 & \& \mathcal{I}_{+1} \\
 & \textcircled{\ominus} \$ \begin{matrix} +1 \\ 1 \end{matrix} \quad +1+ \quad \textcircled{\ominus}^1 \begin{matrix} + \\ 1 \end{matrix} \\
 & (\quad +1) \\
 & \quad \quad \quad \& \quad \quad \quad ' \\
 & \quad \quad \quad +1 \quad \quad \quad +1
 \end{aligned}$$

$$\begin{aligned}
 & \$ (\quad +1 \quad \& \quad ' \quad \textcircled{\ominus} \quad \$ \quad +1 \quad +1+ \quad \textcircled{\oplus}^1 \quad +) \\
 & \quad \quad \quad +1 \quad +1 \quad + \quad 1 \quad +1 \quad +1+ \quad 1 \quad +) \\
 & \textcircled{\ominus} \quad \$ \quad + \quad \textcircled{\ominus} \quad + \\
 & \quad \quad \quad 1 \quad \quad \quad 1 \quad \quad \quad +)
 \end{aligned}$$

\mathcal{I}
■

Proof Lemma 5.

$$\begin{aligned}
 & + \quad + + \# (\quad - \quad +) \\
 & + \quad + + \# (\quad - \quad +) \\
 & ! \quad +1 \quad +1 \quad (\quad +1) \\
 & \quad \quad \quad \# \quad \textcircled{\ominus} \quad \$ \quad + \quad \textcircled{\ominus} \quad + \\
 & \quad \quad \quad \quad \quad \quad 1 \quad \quad \quad 1 \quad \quad \quad +) \\
 & \quad \quad \quad + \quad \quad \quad \# \\
 & + \quad \quad \quad -1 \\
 & -1 \quad -1+ \quad + + \# (\quad -1 \quad - \quad +) \quad + + \# (\quad - \quad +) \quad + \\
 & \quad \quad \quad + \quad \quad \quad ! \quad +1
 \end{aligned}$$

#

! -1

! +1
! -1

-1
! +1

#

-1

-1

% -1 ≤ ! -1 ! +1



0 ! 0 -1 ! -1

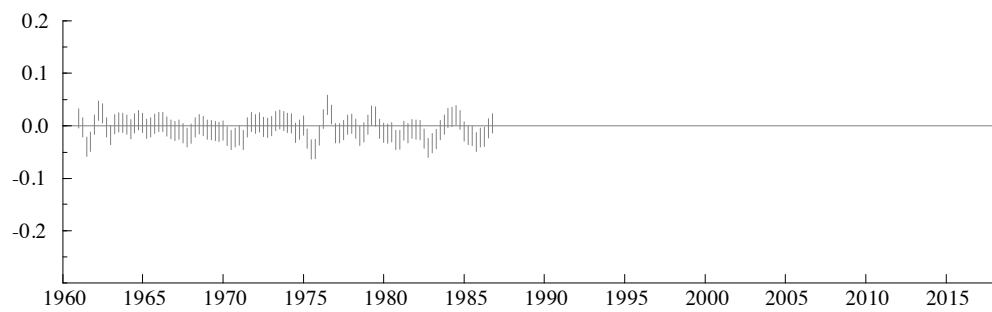
0 30 0 1
0 22
0

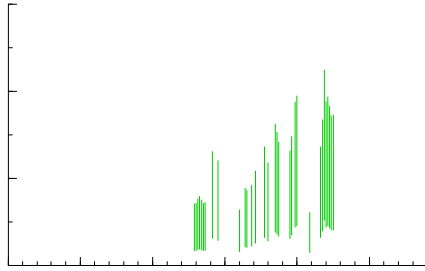
) 0

) 0 0 0

B.3 Empirical Stock-Price Intervals

- + # - + \$ "2 { } 1 2 #
+1 +1 \mathcal{I}
#





B.6 Definition of the Dummy Variables and Subsample Dummy Variables

$$\delta_{126}$$

$$\begin{aligned}
 & 1 \quad 1(\text{200}(\cdot)) \quad 2 \quad 1(\text{200}(1)) \quad 3 \quad 1(\text{200}(2)) \\
 & 4 \quad 1(\text{200}(3)) \quad 5 \quad 1(\text{200}(\cdot)) \quad 6 \quad 1(2010(1) \leq \leq 2010(2)) \\
 & 1(\cdot) \quad 1 \quad (\cdot)
 \end{aligned}$$

$$* + 1 + 2 ,$$

$$0 \quad 0.001$$

$$\begin{aligned}
 & 1(\cdot \leq \leq \cdot_{+1} - 1) \quad 1 \quad 2 \quad 12 \quad \cdot \\
 & \quad 1 \quad (3) \quad 1 \quad (3) \quad 1 \quad 2(1) \quad 2000(\cdot) \quad 2001(2) \quad 2001(\cdot) \quad 2002(2)
 \end{aligned}$$

C The Proxy for the Market Sentiment

$$\frac{t^-}{t+} \quad \frac{t}{t+1}$$